Science, Service, Stewardship

Estimating Biomass and Management Parameters from Length Composition Data: A Stock Assessment Method for Data Deficient Situations

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NOAA FISHERIES SERVICE

Outline

- Motivation
- Model description
- Compare a numbers-based length cohort analysis (Jones LCA, 1979) method to a new biomass-based method that explicitly incorporates growth.
- Investigate the performance of the biomass-based LCA and the more traditional numbers-based LCA on simulated data.
- Demonstrate management applications.
- Test performance by applyingthe model to actual data on eastern Bering Sea northern rock sole

Motivation

- Long time series of catch are not always available.
- Small fish populations are not usually assessed with research surveys.
- Often catch is recorded in weight and by size groups, but no age data are collected.
- FAO (2005) reports that 143 exploited stocks (20%) are not assessed due to lack of available information.
- These situations are exactly those that describe small-scale or artisanal fisheries.
- Stocks still need active management to maintain sustainability.

Objective

- Describe a biomass-based cohort analysis method based on length composition data (LCA) that can be used in small-scale fisheries situations.
- Develop model extensions to allow the calculation of relevant management metrics using only length composition data.
- Apply to data of an exploited and managed stock (i.e. assessments and research surveys performed)

Typical LCA Calculations

Step	Number-based LCA	Biomass-based LCA
1	$CN = \frac{CW}{\overline{W}}$	$C_l^W = CW\ p_l^W$
2	$C_l^N = CN\ p_l^N$	
3	$\hat{N}_l = fxn(C_l^N,\ M,\ K,\ L_\infty)$	$\hat{B}_l = fxn(C_l^W,\ M,\ K,\ L_\infty,\ W_l)$
4	$\hat{B}_l = N_l W_l$	

CN – catch in number

CW – catch in weight

$\overline{W}$ – average weight

N_l – number at length

B_l – biomass at length

p_l^N – proportion of catch in number-at-length

p_l^W – proportion of catch in weight-at-length

C_l^N – catch in number-at-length

C_l^W – catch in weight-at-length

W_l – weight at length

Problems with Numbers-based LCA

- In the Jones numbers-based method, catch weight is converted to numbers, abundance is estimated in numbers, and then population numbers are converted back to weight (biomass) for management actions (i.e. TAC, quota etc).
- The first and last step introduce errors into the population estimates.
- The first and last step can be eliminated by directly using catch that is given in weight-at-length to estimate biomass-at-length.
- Numbers-based methods assume mortality (Z) is the only process affecting biomass. Even if Z=0, growth (G) affects changes in biomass.
- Numbers-based methods will ALWAYS overestimate biomass when growth is positive.

5 Data Requirements

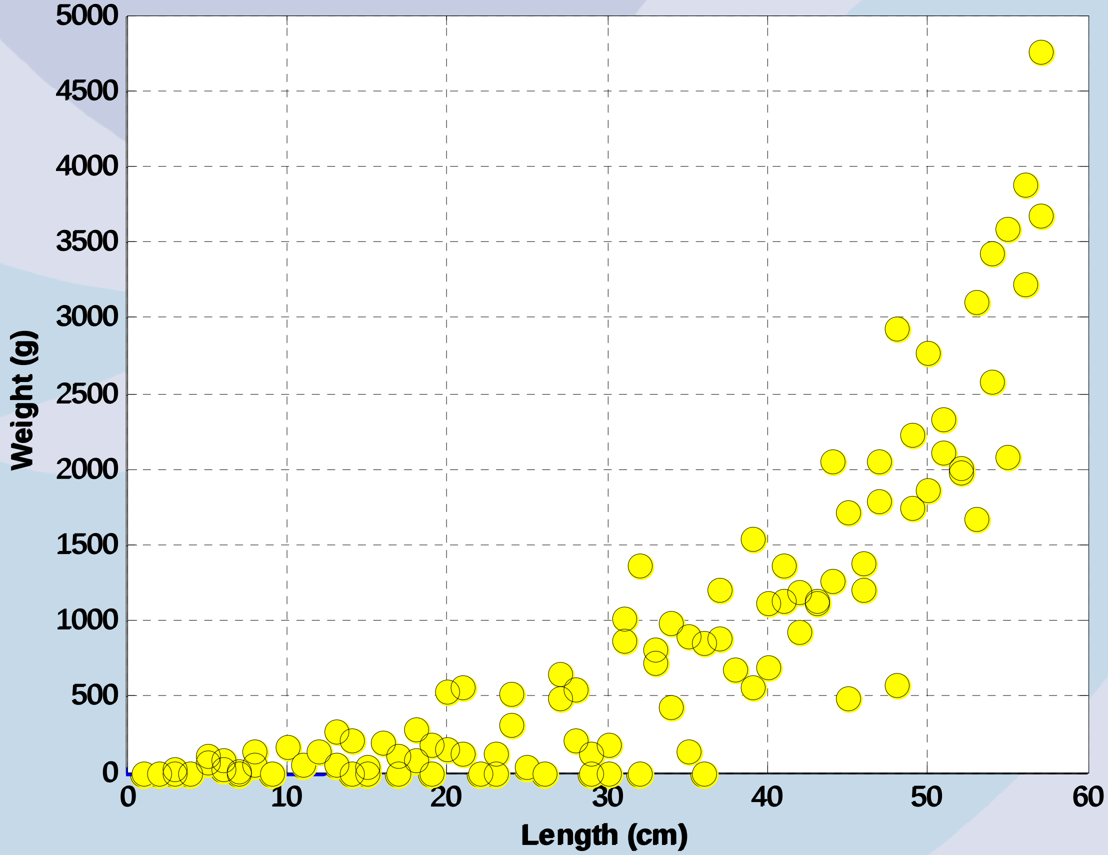
Data Requirements from Fishery

- 1. Length-frequency data. Weight at length. Catch length composition (catch biomass by length interval) for one harvest year minimum.
- 2. Total catch biomass (one harvest year minimum).

Data Requirements-General

- 3. Length-Weight Data
- **parameters: Allometric length-weight parameters (*α, β*)**
- **data: length, weight**

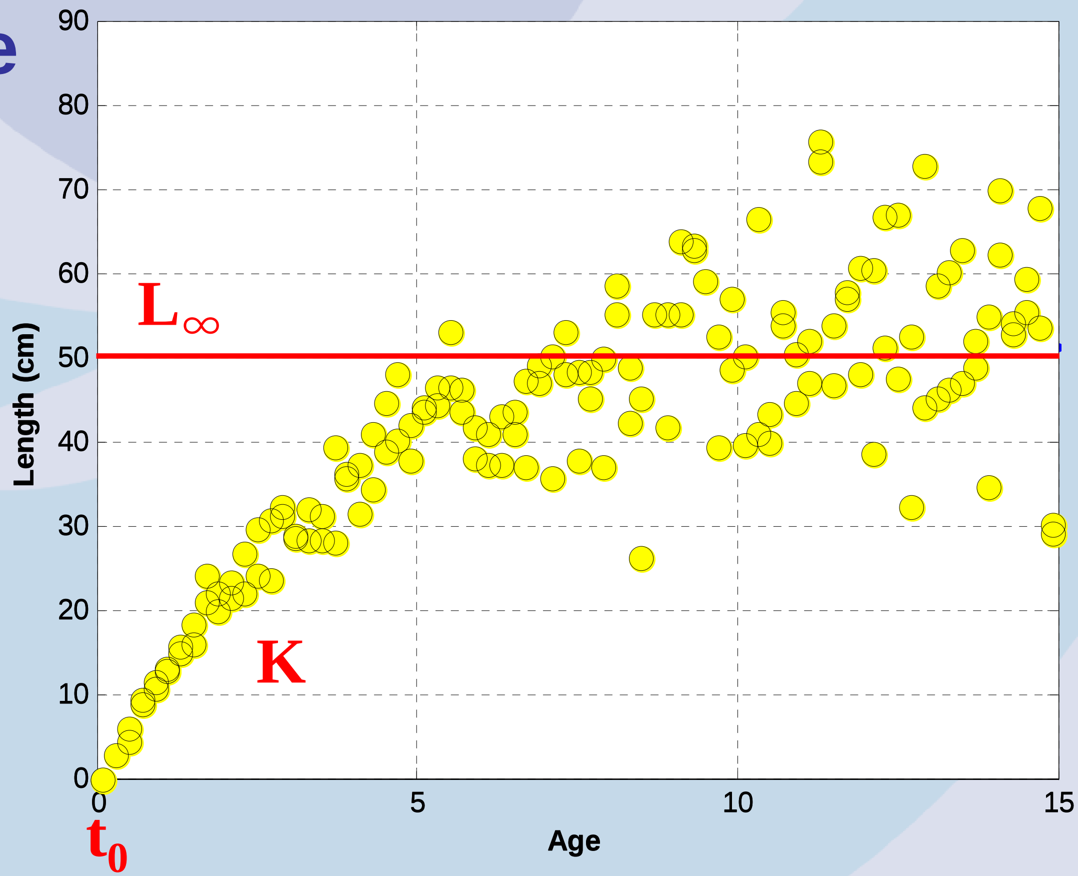
$$W_l = \alpha L_l^{\beta}$$



Data Requirements-General

- 4. Length-at-Age Data
- **parameters: von Bertalanffy parameters (*K*, *L*$_\infty$, *t*$_0$)**
- **data: length, age**

$$L_t = L_\infty(1 - e^{(-K(t-t_0))})$$



Data Requirements

- 5. Natural Mortality (M)
- **Use empirical relationship based on life history parameters**
- C.I Zhang and B.A. Megrey. A revised Alverson and Carney model for estimating the instantaneous rate of natural mortality. 2006. Transactions of the American Fisheries Society 135: 620-633.
- **t_{mb}=fxn(t_{max})**

$$M = \frac{\beta K}{e^{K(t_{mb}-t_0)} - 1}$$

The Model

1. The generalized equation for change in biomass (indexing backwards in time) is

$$B_t = B_{t+\Delta t} \exp(M \cdot \Delta t - G_t) + C_t \exp\left(\frac{M \cdot \Delta t - G_t}{2}\right)$$

2. We can solve the von Bertalanffy growth equation for *t*

$$L_t = L_\infty (1 - e^{(-K(t-t_0))})$$

$$t_{l_i} = t_0 - \frac{1}{K} \ln\left(\frac{L_\infty - l_i}{L_\infty}\right)$$

3. If Δt is the time to grow from length class l_i to length class $l_{i+\Delta l}$ then

$$t_{l_i} = t_0 - \frac{1}{K} \ln\left(\frac{L_\infty - l_i}{L_\infty}\right)$$

$$t_{l_i+\Delta l} = t_0 - \frac{1}{K} \ln\left(\frac{L_\infty - l_{i+\Delta l}}{L_\infty}\right)$$

and

$$\Delta t_{l_i} = t_{l_i+\Delta l} - t_{l_i} = \frac{1}{K} \ln\left(\frac{L_\infty - l_i}{L_\infty - l_{i+\Delta l}}\right)$$

The Model (con't)

4. Substituting 3 into 1 gives

$$B_{l_i} = B_{l_i+\Delta l} \exp\left(\frac{M}{K}\ln\left(\frac{L_\infty - l_i}{L_\infty - l_{i+\Delta l}}\right) - G_{l_i}\right) + C_{l_i} \exp\left(\frac{M}{2K}\ln\left(\frac{L_\infty - l_i}{L_\infty - l_{i+\Delta l}}\right) - \frac{G_{l_i}}{2}\right)$$

5. Which simplifies to

$$B_{l_i} = \left(B_{l_i+\Delta l} X_{l_i} + C_{l_i}\right) X_{l_i}$$

where

$$X_{l_i} = \left(\frac{L_\infty - l_i}{L_\infty - l_{i+\Delta l}}\right)^{\frac{M}{2K}} \times \exp[-\frac{G_{l_i}}{2}]$$

and

$$\Delta t_{l_i} = \frac{1}{K}\ln\left(\frac{L_\infty - l_i}{L_\infty - l_{i+\Delta l}}\right)$$

$$= \exp\left(\frac{M \cdot \Delta t_{l_i} - G_{l_i}}{2}\right)$$

6. Finally, convert length to weight $W_{l_i} = \alpha \; l_i^{\beta}$

7. And calculate growth rate per length class $G_{l_i} = \ln\left(\frac{W_{l_i+\Delta l}}{W_{l_i}}\right)$

LCA-The Steps

Step	Description	Formula
1	Calculate *W* from *l*	$W_{l_i} = \alpha\left(\frac{l_i + l_{i+\Delta l}}{2}\right)^{\beta}$
2	Calculate growth rate *G*	$G_{l_i} = \ln\left(\frac{W_{l_i+\Delta l}}{W_{l_i}}\right)$
3	Calculate Δt_{li}	$\Delta t_{l_i} = \frac{1}{K}\ln\left(\frac{L_\infty - l_{l_i}}{L_\infty - l_{l_i+\Delta l}}\right)$
4	Calculate X_l	$X_{l_i} = \exp\left(\frac{M \cdot \Delta t_{l_i} - G_{l_i}}{2}\right)$
5	Estimate biomass in longest length interval	$B_{l_i} = C_{l_i} \cdot \frac{(M+F)\cdot \Delta t_{l_i} - G_{l_i}}{F \cdot \Delta t_{l_i}}$
6	Recursive equation	$B_{l_i} = (B_{l_i+\Delta l} \cdot X_{l_i} + C_{l_i}) \cdot X_{l_i}$
7	Calculate *F*	$F_{l_i} \cdot \Delta t_{l_i} = \ln\left(\frac{B_{l_i}}{B_{l_i+\Delta l}}\right) - M \cdot \Delta t_{l_i} + G_{l_i}$

Spreadsheet Calculation

Constants

W=aL^b	a	b	L_inf	K	M	F	Z
	0.0018	3.567	51.67	0.299	0.424	0.424	0.848
				M/K =	**1.418**	**F/Z =**	**0.500**

Intermediate Calculations

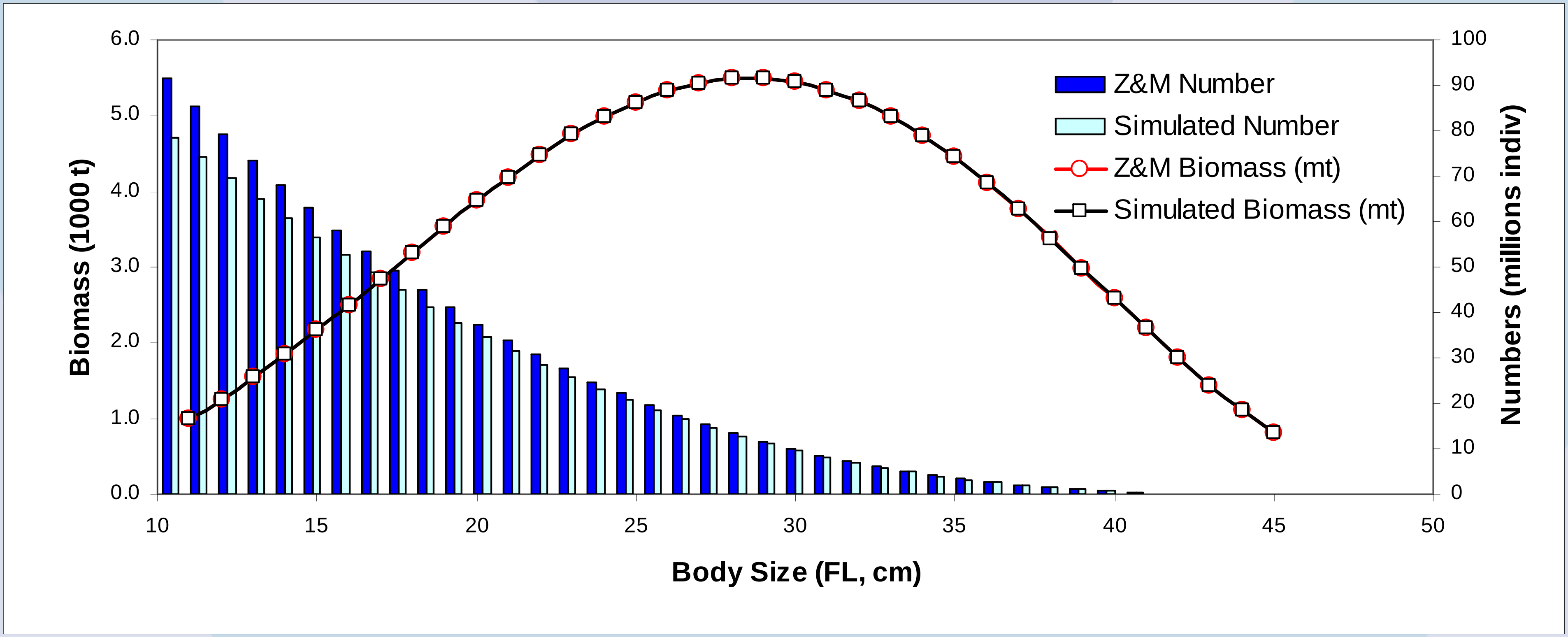
Data

Length(l)	W(l) (g)	C(l)	dt	G(l)	X(l)	B(l)	N(l)	S_t	Z*dt	F/Z	F*dt
12	14.72	12.803	0.085388391	0.27	0.888	24860	1689	0.7326	0.0367	0.012	0.000
13	19.37	128.410	0.087625699	0.25	0.897	31536	1628	0.7440	0.0408	0.090	0.004
14	25.00	143.369	0.08998341	0.24	0.905	39064	1563	0.7563	0.0415	0.080	0.003
15	31.71	373.293	0.092471514	0.22	0.912	47542	1499	0.7638	0.0464	0.155	0.007
16	39.63	1264.125	0.095101136	0.21	0.919	56726	1431	0.7627	0.0610	0.339	0.021
17	48.89	1862.688	0.097884704	0.20	0.925	65832	1347	0.7663	0.0680	0.390	0.027
18	59.60	3455.201	0.100836146	0.19	0.930	74987	1258	0.7601	0.0866	0.506	0.044
19	71.92	4535.111	0.103971121	0.18	0.935	82975	1154	0.7596	0.0965	0.543	0.052
20	85.96	5275.069	0.107307299	0.17	0.940	90051	1048	0.7618	0.1021	0.554	0.057
21	101.88	5712.401	0.110864695	0.16	0.944	96366	946	0.7659	0.1046	0.551	0.058
22	119.81	6572.327	0.114666068	0.16	0.948	102076	852	0.7659	0.1116	0.564	0.063
23	139.92	7811.904	0.118737412	0.15	0.952	106615	762	0.7624	0.1227	0.590	0.072
24	162.34	8053.763	0.123108547	0.14	0.956	109422	674	0.7650	0.1251	0.583	0.073
25	187.24	9424.183	0.127813857	0.14	0.959	111361	595	0.7588	0.1389	0.610	0.085
26	214.78	10078.384	0.132893192	0.13	0.963	111176	518	0.7559	0.1477	0.618	0.091
27	245.12	12703.309	0.138393	0.13	0.966	109461	447	0.7371	0.1776	0.670	0.119
28	278.42	15352.911	0.144367756	0.12	0.970	104101	374	0.7128	0.2155	0.716	0.154
29	314.87	17583.499	0.150881769	0.12	0.973	94903	301	0.6827	0.2627	0.757	0.199
30	354.63	18446.642	0.158011511	0.12	0.976	82189	232	0.6509	0.3143	0.787	0.247
31	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	67342	169	0.6260	0.3569	0.803	0.287
32	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	52686	118	0.6314	0.3517	0.790	0.278
33	[illegible]	[illegible]	[illegible]	0.10	0.987	41295	83	0.5948	0.4145	0.812	0.336
34	550.39	10107.915	0.194841437	0.10	0.990	30299	55	0.5568	0.4837	0.829	0.401
35	609.45	8563.678	0.206898463	0.10	0.994	20684	34	0.4881	0.6182	0.858	0.530
36	672.93	6444.818	0.220546703	0.10	0.999	12308	18	0.3946	0.8335	0.888	0.740
37	741.04	2951.927	0.23612355	0.09	1.003	5890	8	0.4095	0.7988	0.875	0.699
38	813.97	781.570	0.254069085	0.09	1.008	2910	4	0.5975	0.4235	0.746	0.316
39	891.94	197.872	0.274968388	0.09	1.014	2088	2	0.7358	0.2176	0.464	0.101
40	975.13	216.574	0.299616659	0.09	1.020	1836	2	0.7102	0.2552	0.502	0.128
41	1063.78	157.676	0.329122929	0.08	1.028	1552	1	0.7155	0.2498	0.441	0.110
42	1158.07		0.365082002	0.08	1.037	1316	1	0.7884	0.1548	0.000	0.000
43	1258.24	186.874	0.409873496	0.08	1.047	1225	1				
44	1364.50										
						1,782,677	Total Biomass				

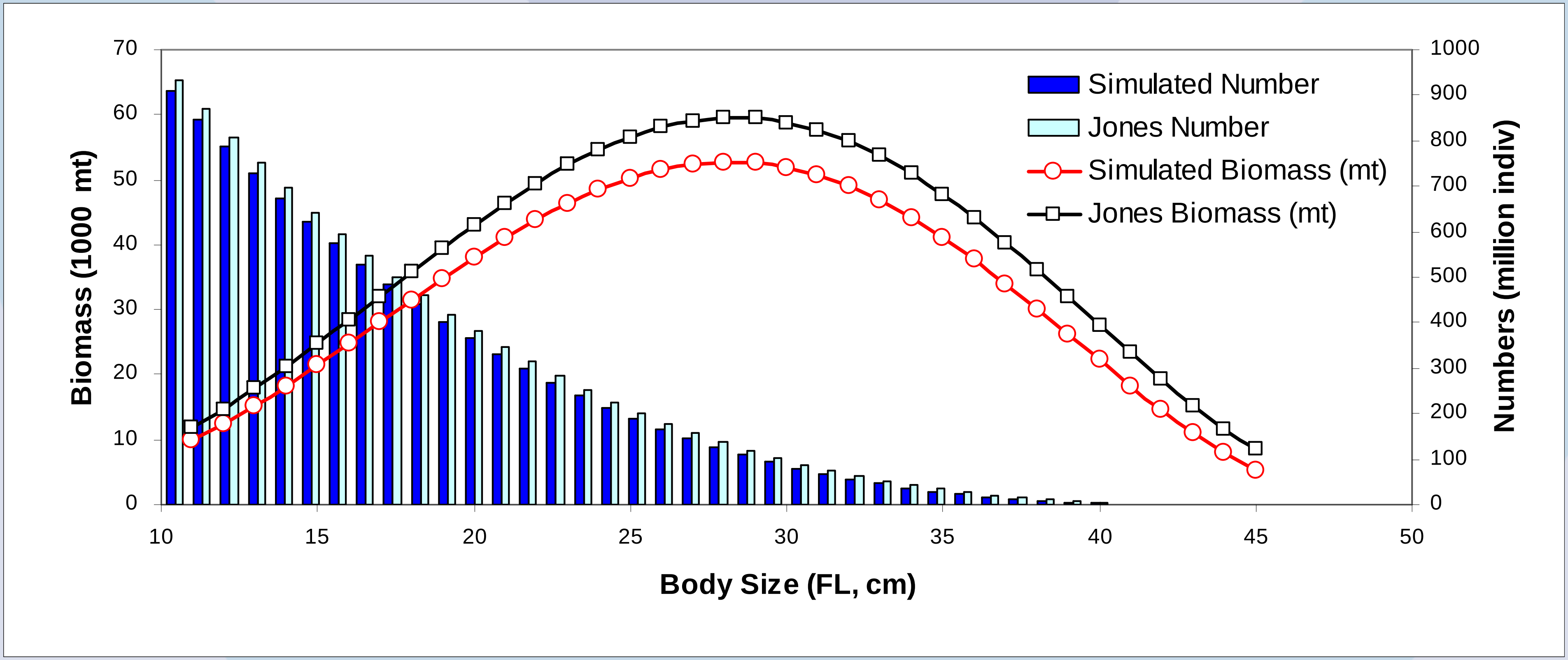
$$B_{l_i} = (B_{l_i+\Delta l} \cdot X_{l_i} + C_{l_i}) \cdot X_{l_i}$$

Comparing Model Performance to Simulated Data

Model Results (based on B) vs. Simulated Data with no error



Jones Model (based on N) vs. Simulated Data with no error



Biomass-based estimation of $F_{x\%}$ Using Length Structure

The fishing mortality ($F_{x\%}$) that maintains the spawning biomass at an arbitrary percentage (x%) of the virgin spawning biomass (i.e. F=0) can be determined by calculating the following ratio.

$$x\% = \frac{Spawning\ Biomass\ with\ \exp loitation\ (F_{x\%})}{Virgin\ Spawning\ Biomass\ (F=0)}$$

Biomass-based estimation of $F_{x\%}$ Using Length Structure

Solving the following equation by using any nonlinear solution algorithm

$$x\% = \frac{\sum_{i=l}^{l_\lambda} B'_i \cdot m_i \cdot e^{G_i - (M + F_{x\%} \cdot S_i) \cdot \left(\frac{1}{K} \ln \left(\frac{(L_\infty - l_i)}{(L_\infty - l_{i+1})} \right) \right)}}{\sum_{i=l}^{l_\lambda} B_i \cdot m_i \cdot e^{G_i - M \cdot \left(\frac{1}{K} \ln \left(\frac{(L_\infty - l_i)}{(L_\infty - l_{i+1})} \right) \right)}}$$

where

$$B_i = B_{i-1} \cdot e^{G_{i-1} - M \cdot \Delta_{i-1}} = B_{i-1} \cdot e^{G_{i-1} - M \cdot \left(\frac{1}{K} \ln \left(\frac{(L_\infty - l_{i-1})}{(L_\infty - l_i)} \right) \right)} \quad for \;\; F = 0$$

$$B'_i = B'_{i-1} \cdot e^{G_{i-1} - (M + F_{x\%} \cdot S_{i-1}) \cdot \left(\frac{1}{K} \ln \left(\frac{(L_\infty - l_{i-1})}{(L_\infty - l_i)} \right) \right)} \quad for \;\; F = x\%$$

$$G_i = \ln \left(\frac{W_{i+1}}{W_i} \right)$$

B_i: Population biomass at length group i when F=0.
B'_i: Population biomass at length group i when $F=F_{x\%}$.
m_i: Maturity ratio of length group i.
l_i: Initial length of length group i.
l_{i+1}: Initial length of length group i+1 (or Maximum length of length group i)
l_λ: last length group.
$F_{x\%}$: Fishing mortality that maintains the spawning biomass at x% of the virgin spawning biomass (or when F=0).
S_i: Fishing selectivity of length group i.
G_i: Growth rate of length group i.

Any Precautionary fishery metric

F_{med}, F_{crit}, F_{lim}, F_{MSY}, $F_{40\%}$, $F_{x\%}$

Estimation of Yield-per Recruit Using Length Structure

$$\frac{Y}{R} = F \cdot W_{\infty} \cdot \exp\left[-M \cdot \left(t_c - t_r\right)\right] \cdot \sum_{n=0}^{3} \frac{U_n \cdot \exp\left[-n \cdot K \cdot \left(t_c - t_0\right)\right]}{F + M + n \cdot K}$$

Yield-per-Recruit Model Using Age Structure

$$\frac{Y}{R} = F \cdot \left(\alpha \cdot L_{\infty}{}^{\beta}\right) \cdot \left[\frac{L_{\infty} - l_r}{L_{\infty}}\right]^{\frac{M}{K}} \cdot \left[\frac{L_{\infty} - l_c}{L_{\infty}}\right]^{\frac{M}{K}} \cdot \sum_{n=0}^{3} \frac{U_n \cdot \left[\frac{L_{\infty} - l_c}{L_{\infty}}\right]^{n}}{F + M + n \cdot K}$$

Yield-per-Recruit Model Using Length Structure

α – length weight coefficient
β – length weight power coefficient
W_{∞} - maximum weight
L_{∞} - maximum length
F – fishing mortality
M – natural mortality
t_c – age of first capture
t_r – age of recruitment
t_0 – von Bertalanffy parameter; t_0 is the theoretical age at which the fish would have length zero if it had always grown as described by the von Bertalanffy equation
K – von Bertalanffy growth rate
l_c – length at first capture
l_r – length at recruitment
U_n – integration coefficient; U_0=1, U_1=-3, U_2=3, and U_3=-1

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$$W_\infty = \alpha \cdot L_\infty^{\ \beta}$$

$$\frac{Y}{R} = F \cdot \left(\alpha \cdot L_\infty^{\ \beta}\right) \cdot \left[\frac{L_\infty - l_r}{L_\infty}\right]^{\frac{M}{K}} \cdot \left[\frac{L_\infty - l_c}{L_\infty}\right]^{\frac{M}{K}} \cdot \sum_{n=0}^{3} \frac{U_n \cdot \left[\frac{L_\infty - l_c}{L_\infty}\right]^n}{F + M + n \cdot K}$$

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$$\exp[-M \cdot (t_c - t_r)] = \exp[-M \cdot (t_0 - t_r)] \cdot \exp[-M \cdot (t_c - t_0)]$$
$$= \left[\frac{L_\infty - l_r}{L_\infty}\right]^{\frac{M}{K}} \cdot \left[\frac{L_\infty - l_c}{L_\infty}\right]^{\frac{M}{K}}$$

$$\frac{Y}{R} = F \cdot (\alpha \cdot L_\infty{}^{\beta}) \cdot \left[\frac{L_\infty - l_r}{L_\infty}\right]^{\frac{M}{K}} \cdot \left[\frac{L_\infty - l_c}{L_\infty}\right]^{\frac{M}{K}} \cdot \sum_{n=0}^{3} \frac{U_n \cdot \left[\frac{L_\infty - l_c}{L_\infty}\right]^{n}}{F + M + n \cdot K}$$

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$$\exp[-n \cdot K \cdot (t_c - t_0)] = \left[\frac{L_\infty - l_c}{L_\infty}\right]^n$$

$$\frac{Y}{R} = F \cdot (\alpha \cdot L_\infty{}^\beta) \cdot \left[\frac{L_\infty - l_r}{L_\infty}\right]^{\frac{M}{K}} \cdot \left[\frac{L_\infty - l_c}{L_\infty}\right]^{\frac{M}{K}} \cdot \sum_{n=0}^{3} \frac{U_n \cdot \left[\frac{L_\infty - l_c}{L_\infty}\right]^n}{F + M + n \cdot K}$$

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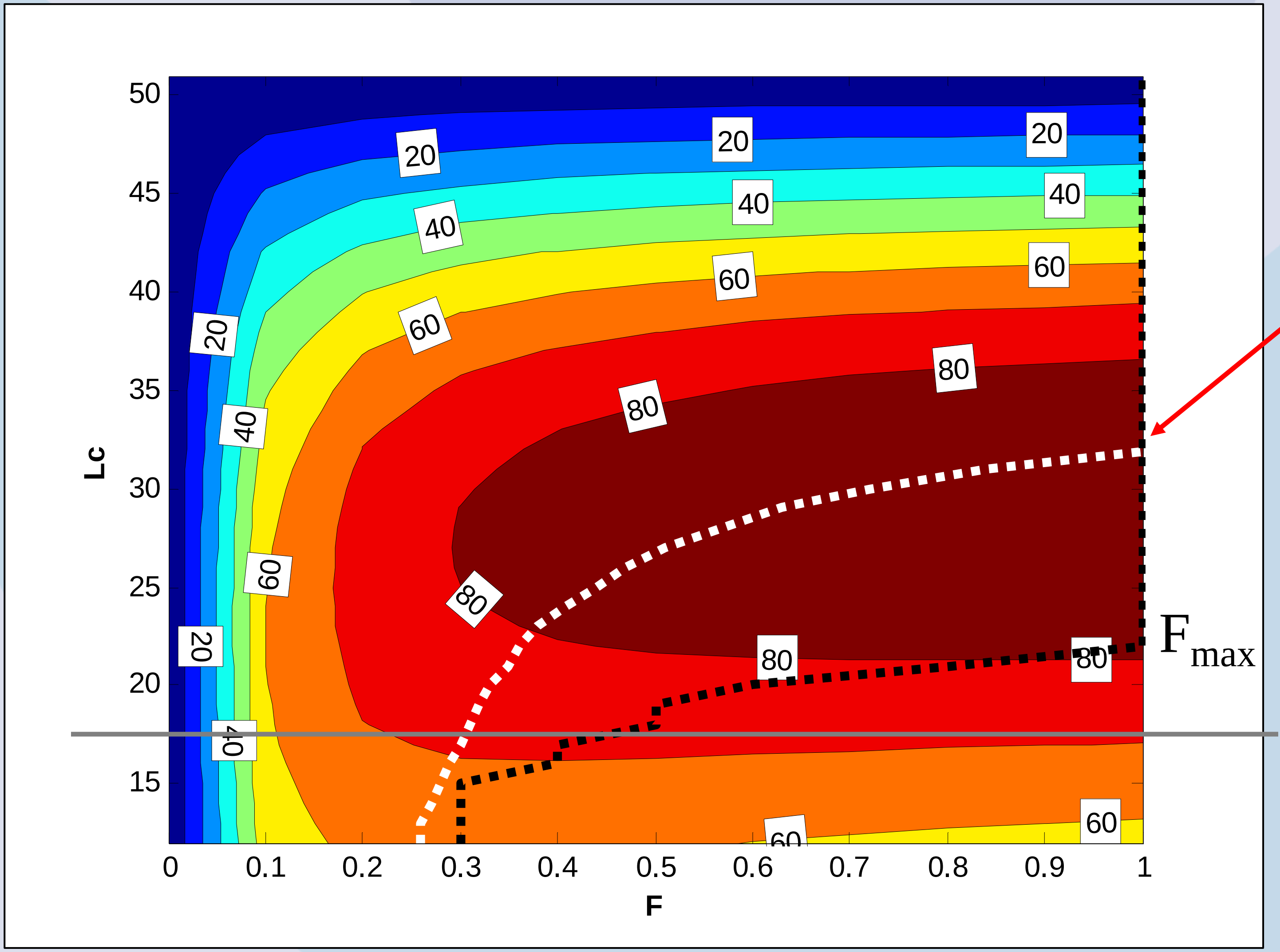
Yield Contours with $F_{40\%}$ and F_{max} Isopleths

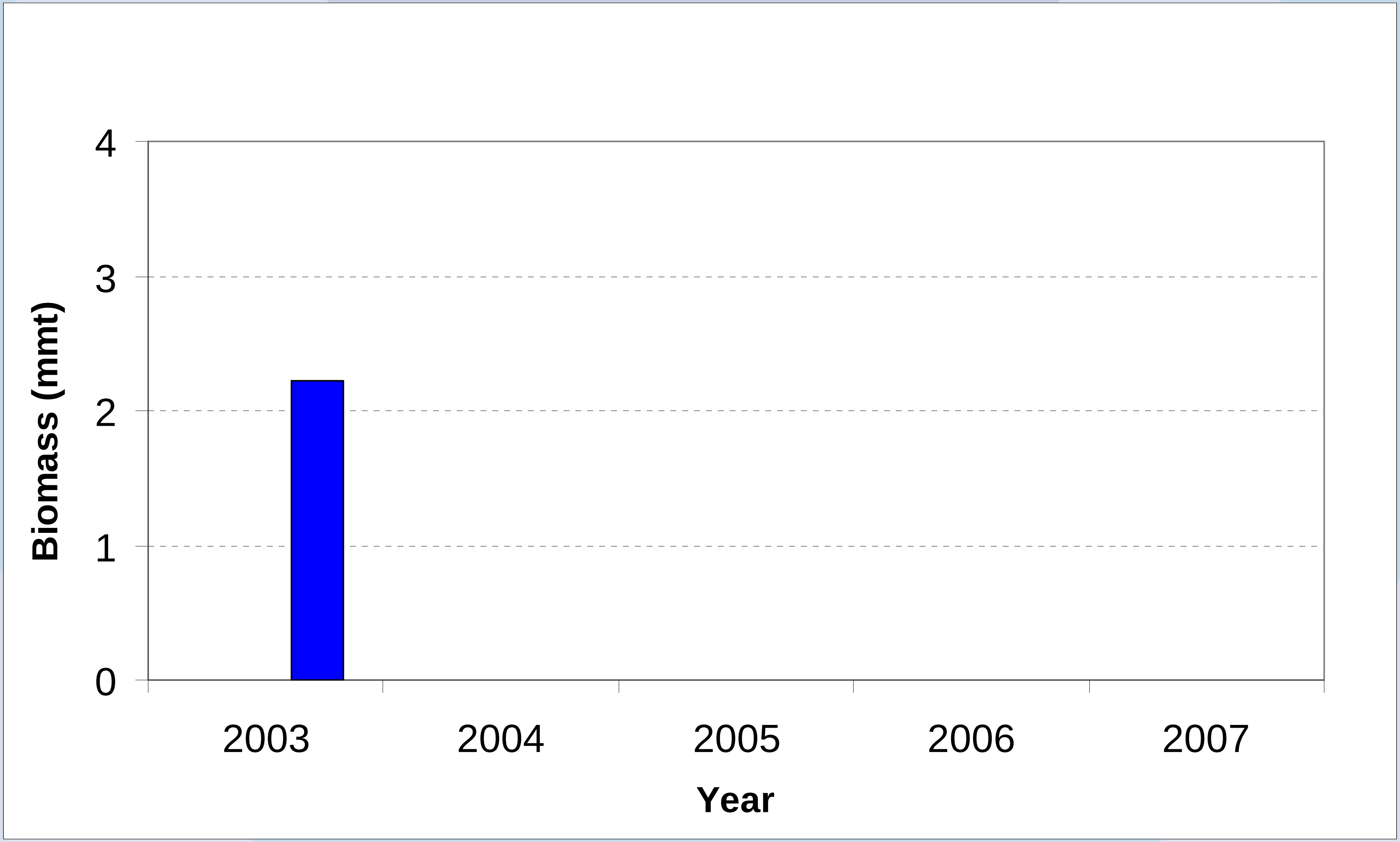


$F_{40\%}$ maximizes yield

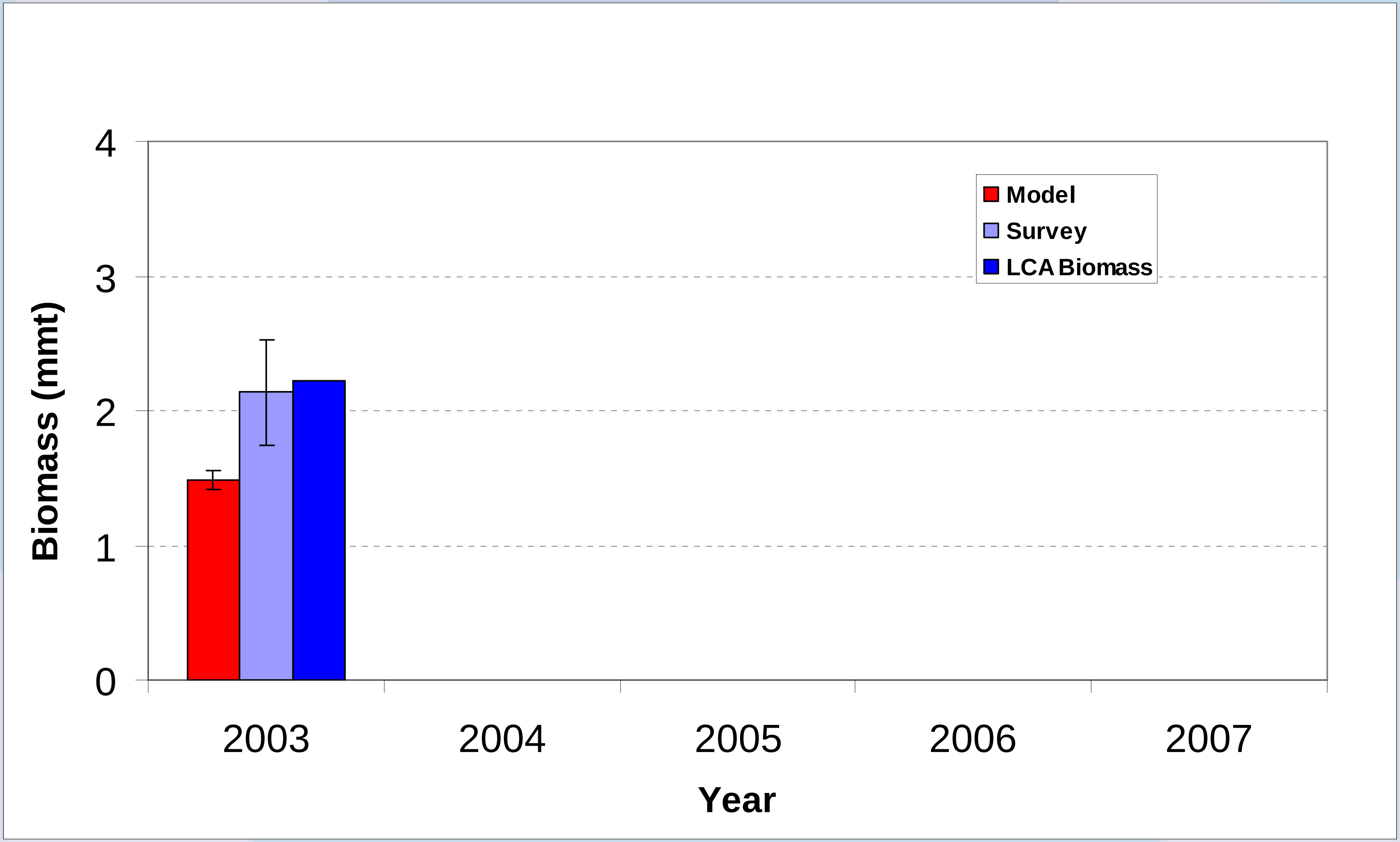
For any length at first capture (L_c), $F_{40\%}$ is always less than F_{max}

Apply model to eastern Bering Sea northern rock sole data

Eastern Bering Sea Northern Rock Sole



Eastern Bering Sea
Northern Rock Sole



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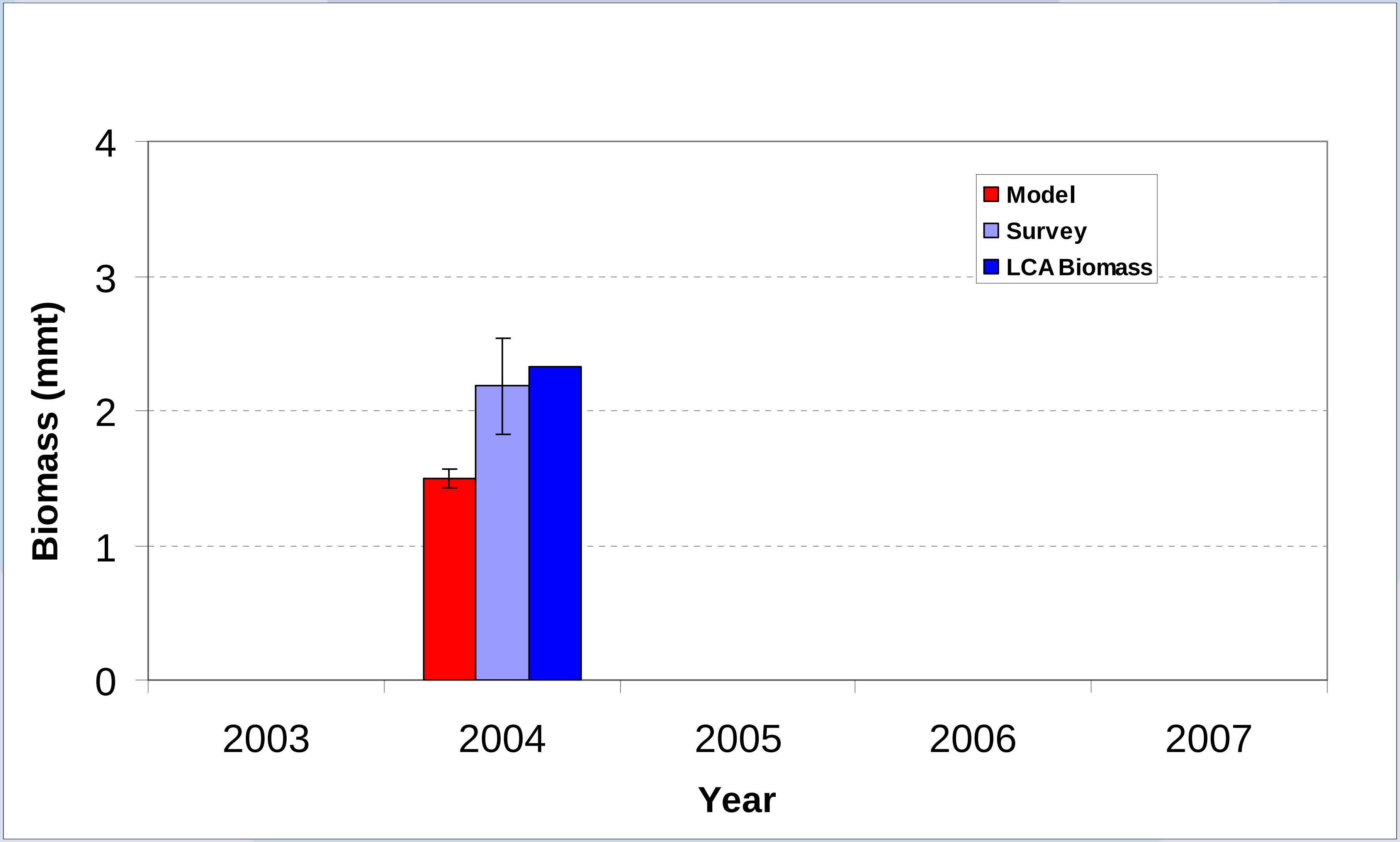

Eastern Bering Sea
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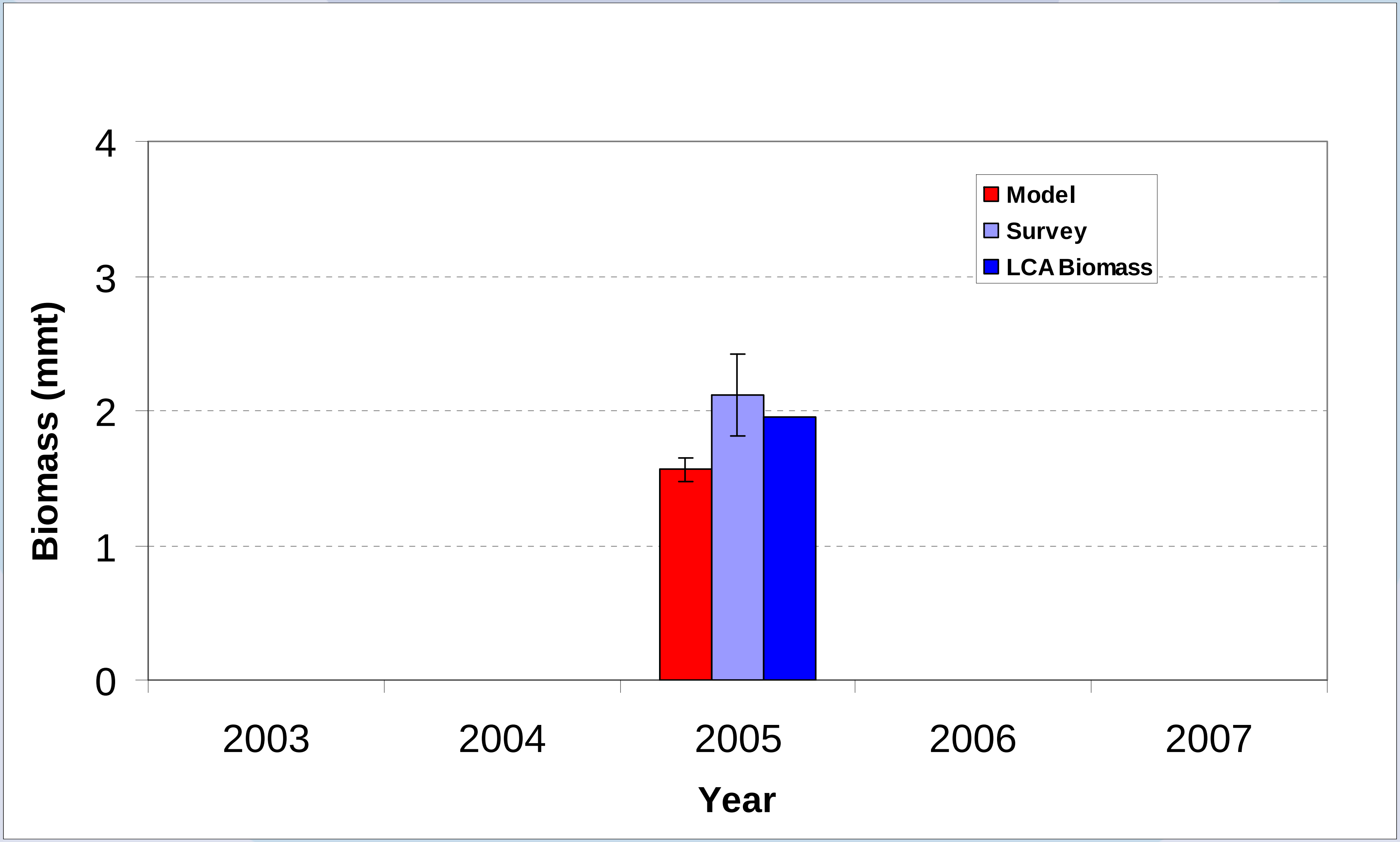
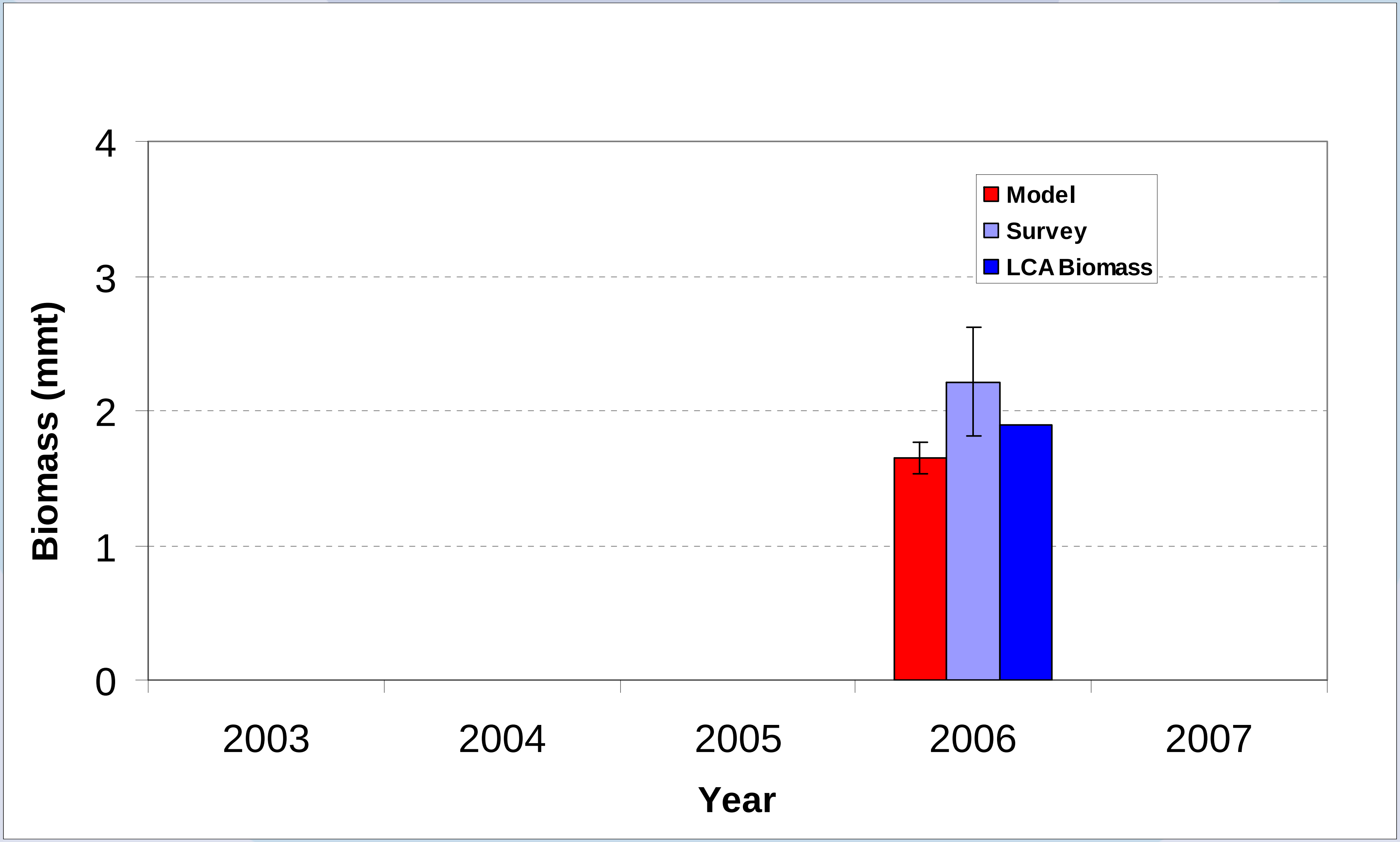


Eastern Bering Sea Northern Rock Sole



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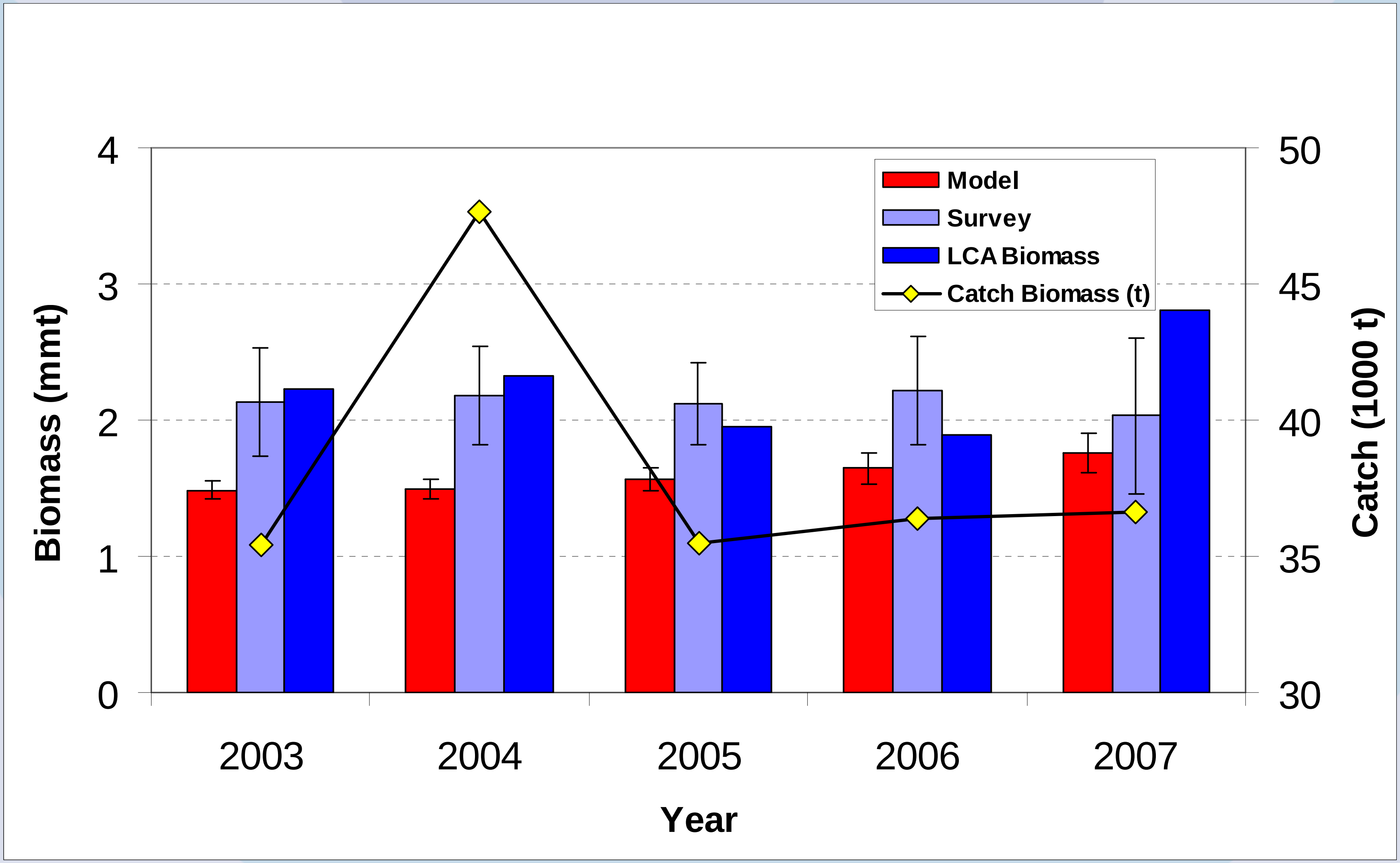
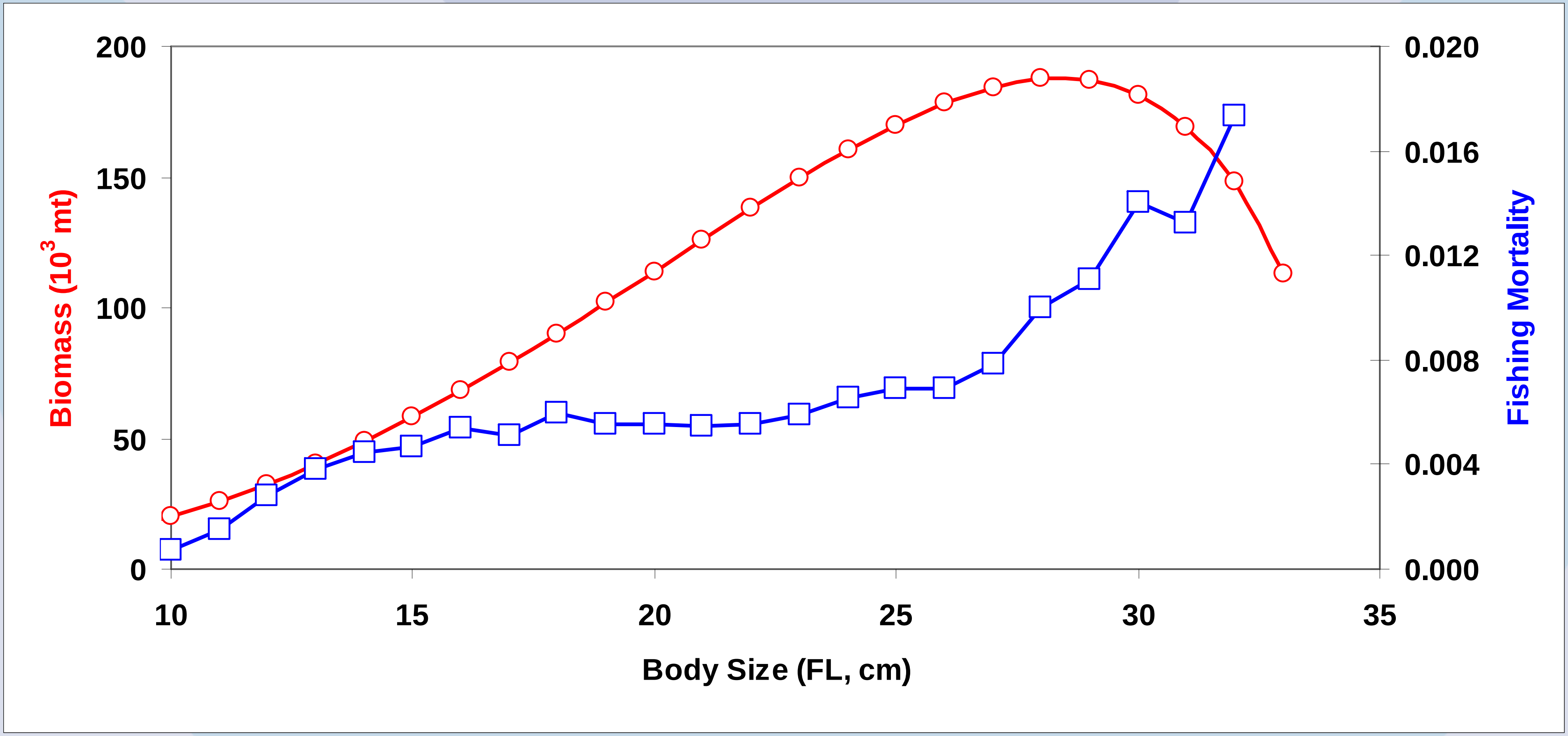


Biomass and Fishing Mortality Northern Rock Sole



Conclusions

- Biomass LCA is simple to apply
  - Assumptions are minimal
  - Calculations are not complicated and easily implemented with spreadsheet software
  - Data needs are modest - only 1 year of catch information
- Method works well compared to simulated data with known properties
- Can be easily extended to include calculation of useful and relevant management metrics
  - Biomass and fishing mortality by length
  - Population biomass
  - $F_{x\%}$ calculations and biomass estimates allow calculation of approximate ABC or TAC
  - Yield per Recruit using length structure
- Biomass LCA should be considered for small scale fisheries resource assessment